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DESIGN THEORY FOR THE PRESSING CHAMBER IN THE SOLID BIOFUEL PRODUCTION PROCESS

Monika Kováčová*, Miloš Matúš, Peter Križan, Juraj Beniak

Slovak University of Technology in Bratislava, Faculty of Mechanical Engineering, Námestie slobody 17, 81231 Bratislava, Slovakia

* corresponding author: monika.kovacova@stuba.sk

Abstract. The quality of a high-grade solid biofuel depends on many factors, which can be divided into three main groups — material, technological and structural. The main focus of this paper is on observing the influence of structural parameters in the biomass densification process. The main goal is to model various options for the geometry of the pressing chamber and the influence of these structural parameters on the quality of the briquettes. We will provide a mathematical description of the whole physical process of densifying a particular material and extruding it through a cylindrical chamber and through a conical chamber. We have used basic mathematical models to represent the pressure process based on the geometry of the chamber. In this paper we try to find the optimized parameters for the geometry of the chamber in order to achieve high briquette quality with minimal energy input.

All these mathematical models allow us to optimize the energy input of the process, to control the final quality of the briquettes and to reduce wear to the chamber. The practical results show that reducing the diameter and the length of the chamber, and the angle of the cone, has a strong influence on the compaction process and, consequently, on the quality of the briquettes. The geometric shape of the chamber also has significant influence on its wear. We will try to offer a more precise explanation of the connections between structural parameters, geometrical shapes and the pressing process. The theory described here can help us to understand the whole process and influence every structural parameter in it.

Keywords: densification process, numerical optimalization for structural parameters, mathematical model for cone chamber, mathematical model for cylindrical chamber.

1. Introduction

Current European legislation set targets for using renewable energy sources, which will result in the gradual replacement of fossil fuels. Biomass is the most promising renewable energy source, and offers the most effective options for energy storing. This leads to a need to carry out research in the area of processing biomass and transforming it into a high-grade solid biofuel. The compaction process can affect the mechanical quality indicators of biofuels, especially their density and mechanical resistance. The geometry of the pressing chamber has an enormous impact on the quality of the briquettes and on the required press pressure. It is therefore appropriate to work on optimizing the geometry of the pressing chamber in order to achieve high briquette quality together with minimum energy input.

2. Structural parameters in the densification process

The quality of solid high-grade biofuels depends on many factors, which can be divided into three groups:

- material parameters,
- technological parameters,
- structural parameters.

The material parameters affecting the quality of briquettes are mostly linked to the characteristics of the starting material (material strength, composition etc.) and some physical constants. The technological parameters (humidity, size of the compression pressure, pressing temperature, pressing speed, etc.) can dramatically affect the process of compaction and the quality of the briquettes. However, the structural parameters have a special place in the pressing process, since the successful production of high-quality briquettes involves synergies between all the groups. The main structural parameters affecting the quality of briquettes are:

- the diameter of the pressing chamber,
- the length of the pressing chamber,
- the convexity of the pressing chamber.

Only a limited amount of work is currently being done on mathematical descriptions of the biomass briquetting and pelleting process, the influence of the parameters of the process on the final quality of the briquettes, and descriptions of the effects of pressure in the pressing chamber. There are no complete mathematical models that deal mainly with the impact of structural parameters on the pressing process. It is clear that a detailed study of the impact of all of structural parameters on the pressing process and the resulting quality of briquettes is a very extensive

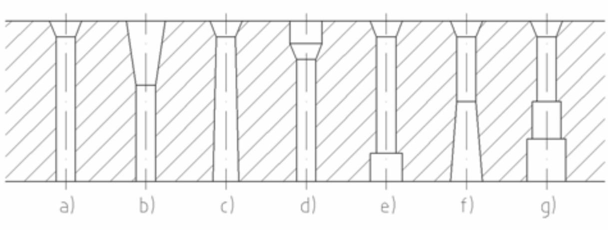

Figure 1. Specification of the geometry of the pressing chambers in the process of pelleting biomass: a) normal, b) deep, c) flat d) well, e) cylindrical, f) conical, g) stepped.

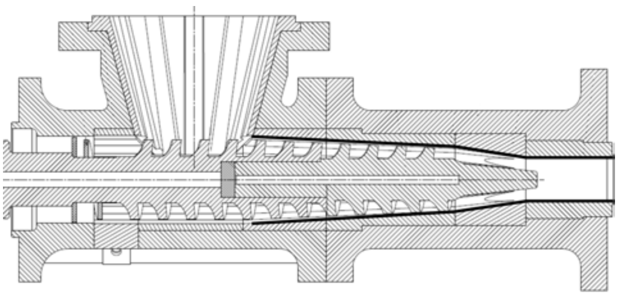

Figure 2. Pressing chamber geometry of the screw briquetting press.

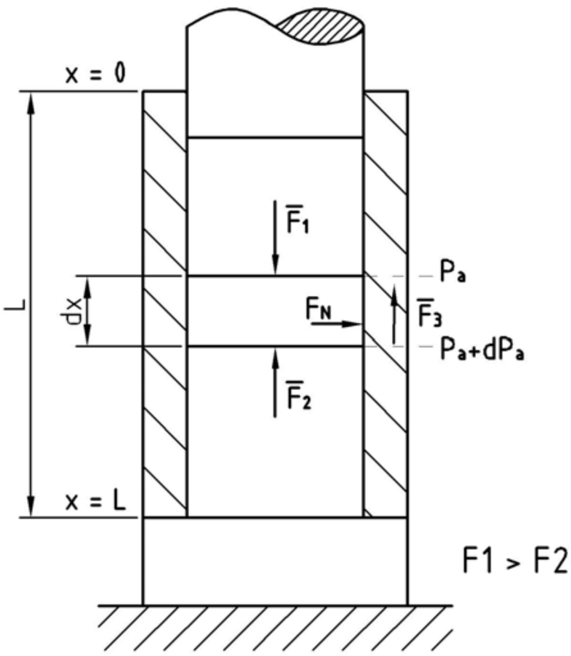

Figure 3. Forces in cylindrical pressing chamber.

undertaking, and requires a detailed analysis of this issue. The most significant influence on the pressing process is from the geometric parameters of the pressing chamber, i.e. the shape and dimensions (diameter, length and convexity of the chamber). The geometry of the pressing chambers currently used for producing solid biofuels is very diverse. It consists of a cylindrical part, in most cases also with a conical part. There is often a combination of several cylindrical and conical parts (Figure 1, Figure 2). The length of the cylindrical part provides the necessary back pressure by the friction part of the force. It also provides biofuels with a high-quality smooth surface. The conical part of the chamber provides spatial movement of the particles and a higher degree of compaction, resulting in higher production quality. When the material is extruded through the conical part of the chamber, the briquettes are given greater density and strength. However, the friction and pressing conditions in the conical chamber greatly increase the required press pressure. The shape and the size of the pressing chamber have a direct impact on the production quality and on the size of the required compression pressure. It is therefore necessary to provide a mathematical description of the whole physical process of densifying a particular material and extruding it through a cylindrical chamber, a conical chamber and also a combined chamber. The mathematical models describing the pressure conditions that are presented here form the basis of our study of the geometry of the chamber. Our study focuses on optimizing the chamber geometry in order to achieve high briquette quality together with minimum energy input.

3. Mathematical background — a cylindrical chamber

We will use the following notation in this paper, see Figure 3:

- dx — height of an infinitesimal cylinder, $dx > 0$;
- d — cylinder diameter;
- S_v — area of the bottom of the cylinder;
- S — surface area of the cylinder;
- p_a — axial pressure
- p_r — radial pressure
- dp_a — the pressure change between the top and bottom base, $dp_a < 0$.

We suppose that $\boldsymbol{F}_1 > \boldsymbol{F}_2$. Based on force equilibrium, we can state the following equation:

$$\boldsymbol{F}_1 - \boldsymbol{F}_2 - \boldsymbol{F}_3 = 0.$$

By simple computation we can state that the force acting on the top base is

$$\boldsymbol{F}_1 = p_a S_v = p_a \pi \Big(\frac{d}{2}\Big)^2 = p_a \frac{\pi d^2}{4},$$

and the force acting on the bottom base is

$$\begin{aligned}\boldsymbol{F}_2 &= (p_a + dp_a) S_v \\ &= (p_a + dp_a)\pi\Big(\frac{d}{2}\Big)^2 = (p_a + dp_a)\frac{\pi d^2}{4}.\end{aligned}$$

The friction force $\boldsymbol{F}_3$ is a special case. We will use the coefficient of support friction and the axial force to evaluate $\boldsymbol{F}_3$:

$$\boldsymbol{F}_3 = \mu F_N = \mu p_e S = \mu p_r \pi d\, dx.$$

We know that the radial pressure and the axial pressure should be connected with their horizontal compacting ratio λ:

$$\lambda = \frac{\sigma_r}{\sigma_a} = \frac{p_r}{p_a},$$

where σ_r is the radial stress and σ_a is the axial stress. So we have

$$\boldsymbol{F}_3 = \mu \boldsymbol{F}_N = \lambda p_a \pi d\, dx.$$

Based on equilibrium of forces, we can derive the differential equation for pressure changes between the two bases of the cylinder:

$$\boldsymbol{F}_1 - \boldsymbol{F}_2 - \boldsymbol{F}_3 = 0,$$
$$p_a \frac{\pi d^2}{4} - (p_a + dp_a)\frac{\pi d^2}{4} - \mu\lambda p_a \pi d\, dx = 0.$$

Let us suppose that $dx \to 0$ and $dp_a \to 0$, then

$$\frac{d}{4}\frac{dp_a}{dx} + \mu\lambda p_a = 0. \tag{1}$$

The axial pressure depends on the place, so we need to locate our cylinder on the axes. Based on this, we are able to rewrite the axial pressure to the function relation

$$\frac{d}{4} p_a'(x) + \mu\lambda p_a(x) = 0.$$

Hence we have a linear differential equation with constant coefficients, and we can find its solution in the form

$$p_a(x) = c_1 e^{-kx},$$

wheree k is a constant given by

$$k = \frac{4\mu\lambda}{d}.$$

The result is in accordance with the physical principle that pressure decreases according to distance from the origin of the coordinate. Based on our coordinate system, we have:

- $x = 0$ — the start position of pressing chamber between compactor and material,
- $x = L$ — the start position of pressing chamber.

Thus we are also able to compute the Cauchy problem with the initial conditions $p_a(x) = p_{ap}$, where p_{ap} is the constant pressure of the compactor on the material throughout the pressing phase:

$$\begin{aligned} p_a(x) &= c_1 e^{-kx} \implies p_a(0) = c_1 e^0 = p_{ap}, \\ p_{ap} &= c_1, \\ p_a(x) &= p_{ap} e^{-\frac{4\mu\lambda}{d} x}. \end{aligned} \tag{2}$$

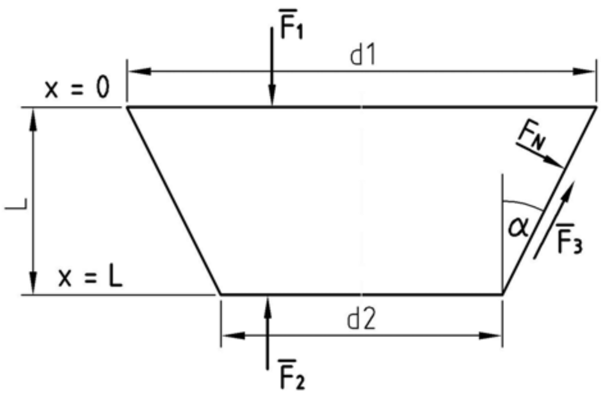

Figure 4. Forces in a conical pressing chamber.

The outgoing pressure on position L can be computed:

$$p_a(L) = p_{ap} e^{-\frac{4\mu\lambda}{d} L}.$$

We are also able to express the incoming pressure p_{ap} in terms of the outgoing pressure:

$$p_{ap} = p_a(L) e^{+\frac{4\mu\lambda}{d} L}.$$

4. Mathematical background — a truncated cone chamber

A truncated cone is a more complicated case than the classical cylinder. Simply speaking, the cylinder is only a special case of the truncated cone, with the elevation angle $\alpha = 0$. We will use the same ideas and the same notation as for the cylinder — see Figure 4.

In the case of a truncated cone, the force equilibrium will change:

$$\boldsymbol{F}_1 - \boldsymbol{F}_2 - \cos\alpha\, \boldsymbol{F}_3 = 0.$$

The direction of friction force $\boldsymbol{F}_3$ contains elevation angle α with the direction of forces $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$. So we can write

$$p_a S_1 - (p_a + dp_a) S_2 - \cos\alpha\, \boldsymbol{F}_3 = 0,$$

where S_1 is the area of the top case and S_2 is the area of the bottom case.

The same coordinate system is used as for the cone.

By simply computation we can state that the force acting on the top base is

$$\boldsymbol{F}_1 = p_a S_1 = p_a \pi \Big(\frac{d_2 + 2v}{2}\Big)^2,$$

where d_2 is the diameter of the bottom case and v is the width of ring of the top case. The force acting on the bottom base is

$$\begin{aligned} \boldsymbol{F}_2 = (p_a + dp_a) S_2 &= (p_a + dp_a)\pi \Big(\frac{d_2}{2}\Big)^2 \\ &= p_a \frac{\pi d_2^2}{4} + dp_a \frac{\pi d_2^2}{4}. \end{aligned}$$

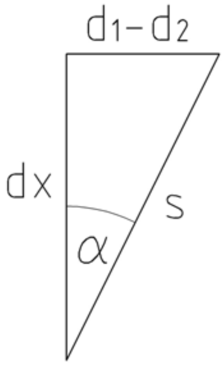

FIGURE 5. Essential dimensions of elementary truncated cone.

Then we have

$$p_a \pi \Big(\frac{d_2+2v}{2}\Big)^2 - p_a \frac{\pi d_2^2}{4} - dp_a \frac{\pi d_2^2}{4} = \boldsymbol{F}_3 \cos\alpha. \quad (3)$$

By simplification we get

$$\boldsymbol{F}_3 = \frac{\pi}{4\cos\alpha}\Big(p_a\big((d_2+2v)^2 - d_2^2\big) - dp_a\, d_2^2\Big) = \mu \boldsymbol{F}_N,$$

where $\boldsymbol{F}_N$ is the normal force.

Now we will try to find a proper evaluation for the friction force. We need first of all to compute the surface area of the cone. We have

$$S_v = \pi\Big(\frac{d_1}{2} + \frac{d_2}{2}\Big)s,$$

where d_1 and d_2 are the diameters of the top and bottom cases of the cone, and s is the length of the lateral surface. Based on Figure 5, we can compute

$$\sin\alpha = \frac{d_1 - d_2}{s} \quad \text{or} \quad \cos\alpha = \frac{dx}{s},$$
$$\text{where} \quad s = \frac{d}{\sin\alpha}.$$

Hence for the surface area of the cone we have

$$S_v = \pi\Big(\frac{d_1}{2} + \frac{d_2}{2}\Big)s = \pi\Big(\frac{d_1}{2} + \frac{d_2}{2}\Big)\frac{dx}{\cos\alpha}.$$

Then

$$\boldsymbol{F}_3 = \mu \boldsymbol{F}_N = \mu p_r \pi\Big(\frac{d_1}{2} + \frac{d_2}{2}\Big)\frac{dx}{\cos\alpha}.$$

We know that the radial pressure and the axial pressure should be connected with their horizontal compacting ratio λ. In the case of a truncated cone, the situation is slightly different. Radial pressure p_r is perpendicular to the lateral surface, so the horizontal ratio must make provision for this:

$$\lambda = \frac{\sigma_r}{\sigma_a}\cos\alpha = \frac{p_r}{p_a}\cos\alpha,$$

where σ_r is radial stress and σ_a is axial stress.

So we have

$$\lambda = \frac{p_r}{p_a}\cos\alpha \quad \text{and} \quad p_r = \frac{\lambda p_a}{\cos_\alpha}.$$

Finally, we have

$$\boldsymbol{F}_3 = \mu \boldsymbol{F}_N = \mu\lambda p_a \frac{1}{\cos\alpha}\pi\Big(\frac{d_2+2v}{2} + \frac{d_2}{2}\Big)\frac{dx}{\cos\alpha}. \quad (4)$$

Let us go back to the equilibrium state equation. From (3) and (4) we have

$$\frac{\pi}{4\cos\alpha}\big(p_a(d_2+2v)^2 - p_a d_2^2 - dp_a\, d_2^2\big) = \mu\lambda p_a \frac{1}{\cos\alpha}\pi\Big(\frac{d_2+2v}{2} + \frac{d_2}{2}\Big)\frac{dx}{\cos\alpha}.$$

By simple computation we have

$$p_a\big((d_2+2v)^2 - d_2^2\big) - dp_a\, s_2^2 = \frac{4\mu\lambda}{\cos\alpha}p_a\Big(\frac{d_2+2v}{2} + \frac{d_2}{2}\Big)dx.$$

We can express the ratio $\tan\alpha = v/dx$. It implies $v = \tan\alpha\, dx$. The left-hand side should be simplified:

$$\begin{aligned} p_a\big((d_2+2v)^2 - d_2^2\big) - dp_a\, d_2^2 &= p_a(d_2^2 + 2vd_2 + 4v^2 - d_2^2) - dp_a\, d_2^2 \\ &= p_a(2\tan\alpha\ dx\, d_2 + 4\tan^2\alpha\, dx^2) - dp_a\, d_2^2. \end{aligned}$$

The infinitesimal element should be considered as sufficiently small, so we can omit the term $4\tan^2\alpha\, dx^2$. Then we have

$$p_a(2\tan\alpha\, dx\, d_2) - dp_a\, d_2^2 = \frac{4\mu\lambda}{\cos\alpha}p_a\Big(\frac{d_2+2v}{2} + \frac{d_2}{2}\Big)dx,$$

$$-dp_a\, d_2^2 = -p_a(2\tan\alpha\, dx\, d_2) + \frac{4\mu\lambda}{\cos\alpha}p_a\Big(\frac{d_2+2v}{2} + \frac{d_2}{2}\Big)dx.$$

Let us suppose that $dx \to 0$ and $dp_a \to 0$, then

$$\frac{dp_a}{dx}d_2^2 = +p_a(2\tan\alpha\, d_2) - \frac{4\mu\lambda}{\cos\alpha}p_a\Big(\frac{d_2+2v}{2} + \frac{d_2}{2}\Big).$$

The axial pressure depends on the place, so we need to locate our cylinder on the axes. Based on this, we are able to rewrite the axial pressure p_a to the function relation

$$\frac{dp_a(x)}{dx}d_2^2 = +p_a(x)(2\tan\alpha\, d_2) - \frac{4\mu\lambda}{\cos\alpha}p_a\Big(\frac{d_2+2v}{2} + \frac{d_2}{2}\Big).$$

Hence we have a linear differential equation with a constant coefficient:

$$d_2^2 p_a'(x) + \bigg(\frac{4\mu\lambda}{\cos\alpha}\Big(\frac{d_2+2v}{2} + \frac{d_2}{2}\Big) - 2\tan\alpha\, d_2\bigg)p_a(x) = 0$$

and we can find its solution in the form

$$p_a(x) = c_1 e^{-kx},$$

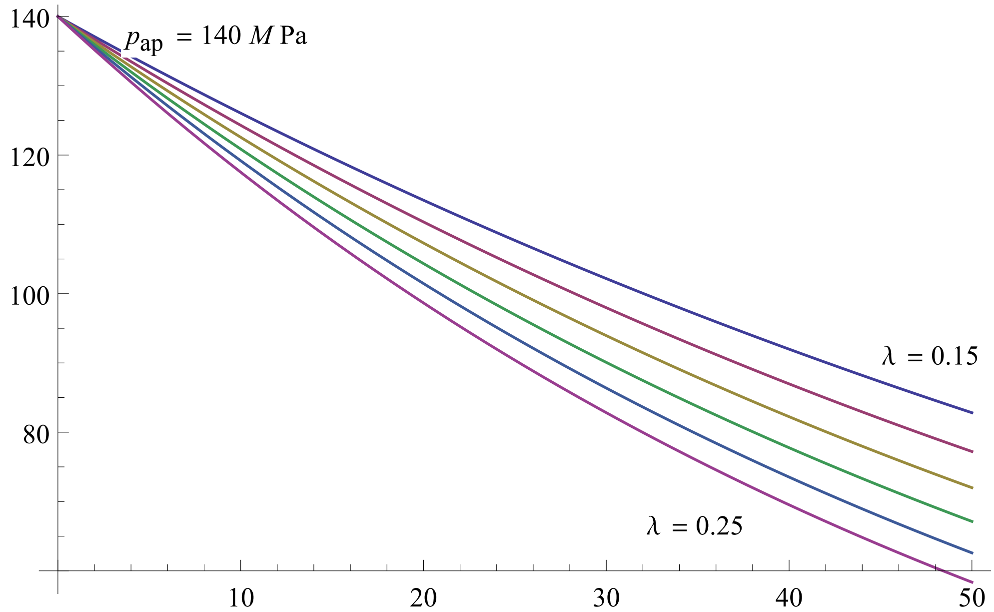

Figure 6. Cylindrical chamber.

where k is a constant given by

$$k = \frac{2\sec\alpha(2d_2\lambda\mu + 2v\lambda\mu - d_2\sin\alpha)}{d_2^2}.$$

The result is in accordance with the physical principle that the pressure decreases according to the distance from the origin of the coordinate. Based on our coordinate system, we have:

- $x = 0$ — start position of pressing chamber,
- $x = L$ — start position of pressing chamber.

Thus we are also able to compute the Cauchy problem with the initial conditions $p_a(x) = p_{ap}$, where p_{ap} is the constant pressure of the compactor on the material during the whole pressing phase:

$$\begin{aligned} p_a(x) &= c_1 e^{-kx} \implies p_a(0) = c_1 e^0 = p_{ap}, \\ p_{ap} &= c_1, \\ p_a(x) &= p_{ap} e^{-\frac{2\sec\alpha(2d_2\lambda\mu+2v\lambda\mu-d_2\sin\alpha)}{d_2^2}x}. \end{aligned} \tag{5}$$

The outgoing pressure on position L can be computed:

$$p_a(L) = p_{ap} e^{-\frac{2\sec\alpha(2d_2\lambda\mu+2v\lambda\mu-d_2\sin\alpha)}{d_2^2}L}.$$

We are also able to express the incoming pressure p_{ap} in terms of the outgoing pressure:

$$p_{ap} = p_a(L) e^{+\frac{2\sec\alpha(2d_2\lambda\mu+2v\lambda\mu-d_2\sin\alpha)}{d_2^2}L}.$$

As was mentioned above, the cylinder is only a special case of the cone, so in the case of angle $\alpha = 0$, the result of (5) should be the same as in (2):

$$\begin{aligned} p_a(x) &= p_{ap} e^{-\frac{2\sec\alpha(2d_2\lambda\mu+2v\lambda\mu-d_2\sin\alpha)}{d_2^2}x} \\ &= p_{ap} e^{-\frac{2\frac{1}{\cos\alpha}(2d_2\lambda\mu+2v\lambda\mu-d_2\sin\alpha)}{d_2^2}x} \\ &= p_{ap} e^{-\frac{2(2d_2\lambda\mu+2v\lambda\mu)}{d_2^2}x} \\ &= p_{ap} e^{-\frac{4\mu\lambda(d_2+2v)}{d_2^2}x} = p_{ap} e^{-\frac{4\mu\lambda d_1}{d_2^2}x}. \end{aligned}$$

If diameters d_1 and d_2 are the same ($d_1 = d_2 = d$), we have

$$p_a(x) = p_{ap} e^{-\frac{4\mu\lambda d}{d^2}x} = p_{ap} e^{-\frac{4\mu\lambda}{d}x}.$$

The outgoing pressure on position L can be computed:

$$p_a(L) = p_{ap} e^{-\frac{2\frac{1}{\cos\alpha}(2d_2\lambda\mu+2v\lambda\mu-d_2\sin\alpha)}{d_2^2}L}.$$

We are also able to express the incoming pressure p_{ap} in terms of the outgoing pressure:

$$p_{ap} = p_a(L) e^{+\frac{2\frac{1}{\cos\alpha}(2d_2\lambda\mu+2v\lambda\mu-d_2\sin\alpha)}{d_2^2}L}$$

5. Numerical experiments

The exact expressions for a conical chamber and for a cylindrical chamber have been described above. Now we will deal with some simple numerical experiments. As has been shown, a linear differential equation was used in both cases for describing the mathematical model. In the case of a cylindrical chamber, the relation between the outgoing pressures on position L should be computed by the expression

$$p_a(L) = p_{ap} e^{-\frac{4\mu\lambda}{d}L}.$$

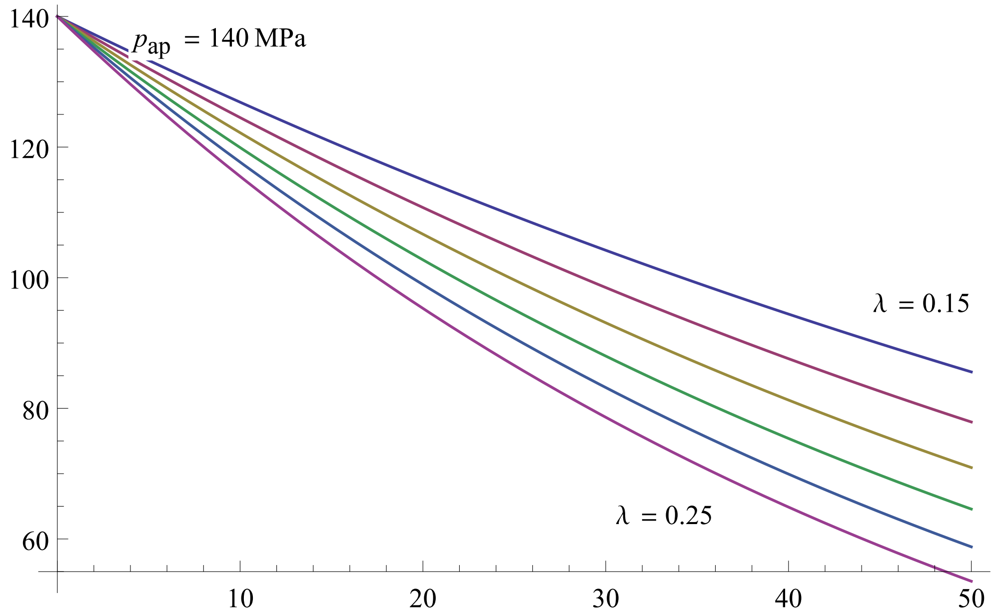

FIGURE 7. Cone case for $\alpha = 2°$.

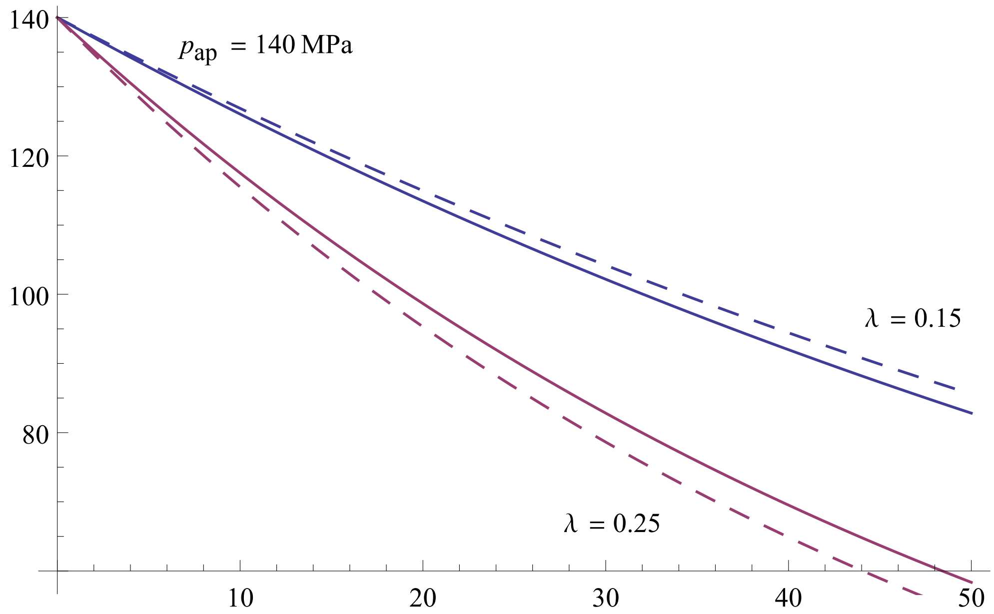

FIGURE 8. Cylindrical shape – solid line, Conical shape for $\alpha = 2°$ – dashed line.

In the case of the cone, the outgoing pressures should be computed by a more complicated expression:

$$p_a(x) = p_{ap} e^{-\frac{2\sec\alpha(2d_2\lambda\mu + 2v\lambda\mu - d_2\sin\alpha)}{d_2^2}L}.$$

Let us take the concrete example of a conical pressing chamber and a cylindrical pressing chamber.

In the case of a cylindrical chamber, we will take $d = 20\,\text{mm}$, $L = 50\,\text{mm}$, $\mu = 0.35$ and λ comes from the range $\lambda \in [0.15, 0.25]$. We will suppose that $p_{ap} = 140\,\text{MPa}$. Then the outgoing press can be modeled by the graph in Figure 6.

For a conical chamber, the situation is more complicated. Let us suppose $\alpha = 2°$ and the length of the pressing chamber is the same $L = 50\,\text{mm}$. Then by simple computation we can set $v = \tan 2° \cdot 50\,\text{mm} = 1.74604$. Let us assume similar conditions as in the previous case $d_1 = d = 20\,\text{mm}$. Then $d_2 + 2v = d_1$ and $d_2 = 20 - 2 \cdot 1.74604 = 16.5079$. The result is in Figure 7.

Simply stated, the shape of the pressure curve remains the same in both cases, but in the case of the cone the outgoing pressure is smaller for some parameters λ than in the case of a cylindrical pressing chamber. We can compare the two cases in one picture. We can see that for $\lambda = 0.15$ the outgoing pressure is greater in a conical shape, but the situation is completely different for $\lambda = 0.25$. In that case, it seems that the conical shape will be more effective. The conical shape is drawn with a dashed line in Figure 8.

6. Conclusions

This paper has presented mathematical models for describing the cylindrical part and the conical part of a pressing chamber. These models form the basis for our whole study in the field of densifying biomass into a solid biofuel. All these mathematical models allow us to optimize the geometry of the pressing chamber and the energy input of the process, to control the final quality of the briquette, and to wear to the chamber. Practical results show that reducing the diameter and the length of the chamber and the angle of the cone have a direct influence on the compacting mechanism and, as a consequence, on the quality of the briquettes. The geometry of the chamber also has a significant influence on its wear. Until now, the geometry of the chamber has been designed mostly empirically, without any research. However, the theory described here can help to understand whole process and influence every structural parameter in the process. The next step in our research leads toward a mathematically optimized chamber geometry together with minimum energy input (minimal pressure).

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